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The Problem of Determining Membership Values in Fuzzy Sets in Real World Situations

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ABSTRACT

One of the fundamental concepts in fuzzy set theory is the one of membership values. An appealing procedure for deriving information about membership values, is to use a matrix of pairwise comparisons [18], [19], [20]. A number of OR approaches that are based on eigenvalue theory and mathematical programming have been proposed to manipulate the previous matrices and estimate membership values. The findings of this paper reveal that although some methods appear to be more effective than others, still their performance is dramatically poor.

1. INTRODUCTION

The more than 1,800 references in [3], [4], and [42] describe the importance of fuzzy set theory in engineering and scientific problems. Fuzzy sets are particularly critical in many decision making situations (see, for example, [1], [7], [8], [9], [10], [15], [16], [21], [22], [25], [26], [27]). Recently, an increasingly large number of AI researchers have been faced by the problem that either their data or their background knowledge is fuzzy. This is a pervasive problem in AI. It is particularly critical to people building expert systems, for the knowledge they are dealing with is almost always riddled with vague concepts and judgmental rules [17]. The impact of fuzzy set theory on AI is best illustrated in [3], [4], and [34] - [43]. The understanding of fuzzy sets is of crucial importance to the successful development and operation of many expert systems (e.g. [11], [12], [16], [41]).

The keystone of any new real life application of fuzzy set theory is the successful estimation of the membership values of the elements in a fuzzy set. Saaty [18], [19], [20] proposed the use of a reciprocal matrix with entries that reflect the decision maker's estimates of the relative importances of the elements of a fuzzy set. In this way, membership values are derived from judgments of human experts about the dominance relations among the elements of a fuzzy set.

There are two main approaches in this decision process. The first approach considers the input data as continuous functions (see for example [26]). The second one, uses discrete data. Although there is no evidence as to why the input data have to be continuous or discrete, the discrete data are much easier to obtain. Saaty [20] claims that decision makers can derive effectively the required data by making a number of pairwise comparisons. In this paper we consider the case of dealing with these pairwise comparisons.

The use of the above reciprocal matrices has captured the interest of many researchers (see, for example, [2], [6], [7], [8], [20], [28], [31]). This is mainly due to the nice mathematical properties of the reciprocal matrices and the fact that the input data are rather easy to be obtained. In this paper we review some of the methods that use the above matrices as input and derive membership values. These methods use Operations Research techniques, namely, eigenvalue theory and mathematical programming. Then, the assumption that in reality membership values take on continuous values is made. This assumption is made in order to capture the majority of the real world cases. Using this assumption a forward error analysis reveals that the tested methods yield dramatically high failure rates.

2. LITERATURE REVIEW

2.1. Reciprocal Matrices with Pairwise Comparisons

Let A_1, A_2,...,A_n be the members of a fuzzy set. We are interested in evaluating the membership values of the above members. Saaty [18], [19], and [20] proposed to use a matrix A of rational numbers taken from the set {1/9, 1/8, 1/7, ..., 1, 2, 3, ..., 7, 8, 9}. Each entry of the above matrix A represents a pairwise judgment. Specifically, the entry a_{ij} denotes the number that estimates the relative membership of element A_i when it is compared with element A_j. Obviously, $a_{ij} = 1/a_{ji}$ and $a_{ii} = 1$. That is, the matrix is a reciprocal one.

Let us first examine the case in which it is possible to have perfect values a_{ij}. In this case it is $a_{ij} = W_i/W_j$ (W_s denotes the actual value of element s) and the previous reciprocal matrix A is consistent. That is:

$$a_{ij} = a_{ik}a_{kj} \quad (i,j,k = 1,2,3,...,n, \text{ where n is the number of elements in the fuzzy set}) \tag{1}$$

It can be proved that A has rank 1 with $\lambda = n$ to be its nonzero eigenvalue. Then we have:

$$Ax = nx \quad \text{where x is an eigenvector} \tag{2}$$

From the fact that $a_{ij} = W_i/W_j$ the following are obtained:

$$\sum_{j=1}^{n} a_{ij} W_j = \sum_{j=1}^{n} W_i = nW_i \qquad i = 1,2,3,...n \tag{3}$$

or:

$$A\,W = n\,W \tag{4}$$

Equation (4) states that n is an eigenvalue of A with W a corresponding eigenvector. The same equation also states that in the perfectly consistent case (i.e., $a_{ij} = a_{ik}\, a_{kj}$) the vector W, with the membership values of the elements 1,2,3,...,n, is the principal right-eigenvector (after normalization) of the matrix A.

2.2. The Eigenvalue Approach

In the non-consistent case (which is the most common) the pairwise comparisons are not perfect, that is, the entry a_{ij} might deviate from the real ratio W_i/W_j (i.e., from the ratio of the real membership values W_i and W_j). In this case, the previous expression (1) does not hold for all the possible combinations. Now the new matrix A can be considered as a perturbation of the previous consistent case. When the entries a_{ij} change slightly, then the eigenvalues change in a similar fashion [20]. Moreover, the maximum eigenvalue is close to n (greater than n) while the remaining eigenvalues are close to zero. Thus, in order to find the membership values in the non-consistent case, one should find an eigenvector that corresponds to the maximum eigenvalue λ_{max}. That is to say, to find the principal right-eigenvector W that satisfies:

$$A\,W = \lambda_{max} W \quad \text{where:} \quad \lambda_{max} \approx n$$

Saaty estimates the reciprocal right-eigenvector W by multiplying the entries in each row of matrix A together and taking the n^{th} root (n is the number of the elements in the fuzzy set). Since we desire to have values that add up to 1.00 we normalize the previously found vector by the sum of the above values. If we want to have the element with the highest value to have membership value equal to 1.00, we divide the previously found vector by the highest value.

Under the assumption of total consistency, if the judgments are gamma distributed (something that Saaty assumes is the case), the principal right-eigenvector of the resultant reciprocal matrix A is Dirichlet distributed. If the assumption of total consistency is relaxed, then Vargas [31] states that the hypothesis that the principal right-eigenvector follows a Dirichlet distribution is accepted if the consistency ratio is 0.10 or less.

The consistency ratio (CR) is obtained by first estimating λ_{max}. Saaty estimates λ_{max} by adding the columns of matrix A and then multiplying the resulting vector with the vector W. Then he uses what he calls the consistency index (CI) of the matrix A. He defines CI as follows:

$CI = (\lambda_{max} - n) / (n-1)$

Then, the consistency ratio CR is obtained by dividing the CI by the Random Consistency index (RC) as given in the following table:

Table 1. Random consistency indices.

n	1	2	3	4	5	6	7	8	9
Random Consistency index (RC)	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Each RC is an average random consistency index derived from a sample of size 500 of randomly generated reciprocal matrices with entries from the set {1/9, 1/8, 1/7, ..., 1, 2, 3, ..., 7, 8, 9} to see if its CI is 0.10 or less.

2.3. Some Minimization Approaches

If the previous approach yields a CR greater than 0.10 then a re-examination of the pairwise judgments is recommended until a CR less than or equal to 0.10 is achieved.

Chu [2] observed that given the data a_{ij} the values W_i to be estimated are desired to have the property:

$$a_{ij} \approx W_i/W_j \tag{5}$$

This is true since a_{ij} is meant to be the estimation of the ratio W_i/W_j. Then, in order to get the estimates for the W_i given the data a_{ij}, they propose the following constrained optimization problem:

$$\text{minimize } S = \sum_{i=1}^{n}\sum_{j=1}^{n}(a_{ij}W_j - W_i)^2 \tag{6}$$

subject to:

$$\sum_{i=1}^{n} W_i = 1$$

$$W_i > 0 \qquad (i = 1,2,3,\ldots,n)$$

They also give an alternative expression, S_1, that is more difficult to solve numerically. That is,

$$S_1 = \sum_{i=1}^{n}\sum_{j=1}^{n}(a_{ij} - W_j/W_i)^2 \tag{7}$$

Federov, et al., in [6] proposed a variation of the above least squares formulation. For the case of only one decision maker they recommend the following models:

$$\ln a_{ij} = \ln W_i - \ln W_j + \Psi_1(W_i, W_j)\,\varepsilon_{ij} \qquad (8)$$

$$a_{ij} = (W_i/W_j) + \Psi_2(W_i, W_j)\,\varepsilon_{ij} \qquad (9)$$

Here W_i and W_j are the true (and unknown) membership values; $\Psi_1(X,Z)$, $\Psi_2(X,Z)$ are given positive functions (when X, Z > 0). The random errors ε_{ij} are assumed independent with zero mean and variance one. Using these two assumptions they are able to calculate the variance of each individual estimated membership value. However, they do not propose any way of selecting the appropriate positive functions.

2.4. The Human Rationality Factor

According to the Human Rationality Assumption (given by Triantaphyllou, et al., in [28]) the decision maker is a rational person. Rational persons are defined here as individuals who try to minimize their regret [23], to minimize losses, or to maximize profit [33]. In the membership evaluation problem, minimization of regret, losses, or maximization of profit could be interpreted as the effort of the decision maker to minimize the errors involved in the pairwise comparisons.

As it is stated in previous paragraphs, in the inconsistent case the entry a_{ij} of the matrix A is an estimation of the real ratio W_i/W_j. Since it is an estimation, the following is true:

$$a_{ij} = (W_i/W_j)d_{ij} \qquad i,j = 1,2,3,\ldots,n \qquad (10)$$

In the above relation d_{ij} denotes the deviation of a_{ij} from being an accurate judgment. Obviously, if $d_{ij} = 1$ then the a_{ij} was perfectly estimated. From the previous formulation we conclude that the errors involved in these pairwise comparisons are given by:

$$\varepsilon_{ij} = d_{ij} - 1.00$$

or (using (10), above):

$$\varepsilon_{ij} = a_{ij}(W_j/W_i) - 1.00 \qquad (11)$$

When a fuzzy set contains n elements, then Saaty's method requires the estimation of the following n(n-1)/2 pairwise comparisons:

$$\begin{array}{lllll} (W_2/W_1) & (W_3/W_1) & (W_4/W_1) & \ldots & (W_n/W_1) \\ & (W_3/W_2) & (W_4/W_2) & \ldots & (W_n/W_2) \\ & & (W_4/W_3) & \ldots & (W_n/W_3) \\ & & & & \vdots \\ & & & & (W_{n-1}/W_n) \end{array} \qquad (12)$$

The corresponding n(n-1)/2 errors are (using relations (11) and (12)):

$$\varepsilon_{ij} = a_{ij}(W_j/W_i) - 1.00 \qquad (13)$$

where: $i,j = 1,2,\ldots,n$
and $j > i$

Since the W_i's are degrees of membership that add up to 1.00 the following relation (14) should also be satisfied:

$$\sum_{i=1}^{n} W_i = 1.00 \tag{14}$$

Apparently, since the W_i's represent degrees of membership we also have: $W_i > 0$ ($i = 1,2,3,\ldots, n$).

Relations (13) and (14), when the data are consistent (i.e., all the errors equal to zero), can be written as follows:

$$B\,W = b \tag{15}$$

Above, the vector b has zero entries everywhere except the last one that is equal to 1.00, and the matrix B has the following form (blank entries represent zeros):

$$
B = \begin{array}{c}
\begin{array}{cccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & \ldots & n-1 & n
\end{array} \\
\left[\begin{array}{cccccccccc}
-1 & a_{1,2} & & & & & & & & \\
-1 & & a_{1,3} & & & & & & & \\
-1 & & & a_{1,4} & & & & & & \\
\cdot & & & & \ddots & & & & & \\
\cdot & & & & & & & & & \\
\cdot & & & & & & & & & \\
\cdot & & & & & & & & a_{1,n-1} & \\
-1 & & & & & & & & & a_{1,n} \\
 & -1 & a_{2,3} & & & & & & & \\
 & -1 & & a_{2,4} & & & & & & \\
 & -1 & & & a_{2,5} & \ddots & & & & \\
 & \cdot & & & & & & & & \\
 & \cdot & & & & & & & & \\
 & \cdot & & & & & & & a_{2,n-1} & \\
 & -1 & & & & & & & & a_{2,n} \\
 & & & & & & & & & \\
 & & & & & & & & -1 & a_{n-1,n} \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & \ldots & 1 & 1
\end{array}\right]
\end{array}
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ n-1 \\ 1 \\ 2 \\ 3 \\ \cdot \\ \cdot \\ \cdot \\ n-2 \\ \cdot\,\cdot\,\cdot \\ 1 \\ \\
\end{array}
$$

The error minimization issue is interpreted in many cases (Regression Analysis, Linear Least Squares problem) as the minimization of the sum of the squared errors [24].

In terms of the formulation (15) this means that in a real life situation (i.e., errors are not zero any more) the real intention of the decision maker is to minimize the expression:

$$f^2(x) = ||b - B\,W||_2^2 \tag{16}$$

which is, a typical Linear Least Squares problem!

If we use the notation described previously then the quantity (6) that is minimized in Chu [2] becomes:

$$S = \sum_{i=1}^{n}\sum_{j=1}^{n}(a_{ij}W_j - W_i)^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}(\varepsilon_{ij}W_i)^2$$

and the alternative expression (7) becomes:

$$S_1 = \sum_{i=1}^{n}\sum_{j=1}^{n}(a_{ij} - W_j/W_i)^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}(\varepsilon_{ij}(W_i/W_j)^2$$

Clearly, both expressions are too complicated to reflect in a reasonable way the intentions of the decision maker.

The models proposed by Federov [6] are closer to the one developed under the Human Rationality Assumption. The only difference is that instead of the relations:

$$\ln a_{ij} = \ln W_i - \ln W_j + \Psi_1(W_i, W_j)\,\varepsilon_{ij}$$

$$a_{ij} = (W_i/W_j) + \Psi_2(W_i, W_j)\,\varepsilon_{ij}$$

the following simpler expression is used:

$$a_{ij} = (W_i/W_j)\,d_{ij}$$

or :

$$a_{ij} = (W_i/W_j)(\varepsilon_{ij} + 1.00) \tag{17}$$

However as it is illustrated in [28], the performance of this method is greatly dependent on the selection of the $\Psi_1(X,Z)$ or $\Psi_2(X,Z)$ functions and now these functions are further modified by (17).

2.5. The Concept of the Closest Discrete Pairwise Matrix

The following forward error analysis is based on the assumption that in the real world the membership values in a fuzzy set take on continuous values. This assumption is a reasonable one since it attempts to capture the majority of the real world cases.

Let $\omega_1, \omega_2, \omega_3, \ldots, \omega_n$ be the real (and thus unknown) membership values of a fuzzy set with n members. If the decision maker knew the above real values then he would be able to have constructed a matrix with the real pairwise comparisons. In this matrix, say matrix A, the entries are given by: $\alpha_{ij} = \omega_i/\omega_j$. This matrix is called the Real Continuous Pairwise matrix, or the RCP matrix [29]. Since in the real world the ω_i's are unknown so are the entries α_{ij} of the previous matrix. However, we can assume here that the decision maker instead of the entries α_{ij} is able to determine the closest values taken from the set {1/9, 1/8, 1/7, ..., 1, 2, 3, ..., 7, 8, 9}. That is, instead of the real (and unknown) value α_{ij} one is able to determine a_{ij} such that:

$$a_{ij} = \min \{|a_{ij} - x/y|\}$$

$$\text{where: } x \in \{1/9, 1/8, 1/7, ..., 1, 2, 3, ..., 7, 8, 9\}$$

In other words, one's judgments about the values of the pairwise comparisons of the i^{th} element when it is compared with the j^{th} one, is so accurate that in real life is the closest (in absolute value terms) to the values one is supposed to choose from. Apparently, this assumption favors the values of the failure rates derived in this paper. This fact indicates that even the failure rates in this paper are smaller than the actual ones.

The matrix with entries the a_{ij}'s that we assume the decision maker is able to construct, has entries from the discrete and finite set {1/9, 1/8, 1/7, ..., 1, 2, 3, ..., 7, 8, 9} This matrix is called the Closest Discrete Pairwise matrix or the CDP matrix.

The following example illustrates a case where we deal with a fuzzy set with three members. The real membership values are assumed to be known. Then, the RCP and CDP matrices are derived. When the error minimization method is applied on this problem the resulted membership values are significantly different than the real ones. More importantly, the ranking is altered too, indicating a case where the error minimization method fails.

EXAMPLE

Let us assume that the real (and unknown) membership values, after normalization, of a fuzzy set with three members are: $\omega_1 = 0.73207 \omega_2 = 0.13366$, and $\omega_3 = 0.13427$

Obviously, the corresponding ranking by magnitude of these members is: $\rho_1 = 1$, $\rho_2 = 3$, and $\rho_3 = 2$.

Then, the RCP (and unknown) matrix with the pairwise comparisons is:

$$\text{RCP} = \begin{bmatrix} 1 & 5.47735 & 5.45216 \\ 0.18257 & 1 & 0.99540 \\ 0.18341 & 1.00462 & 1 \end{bmatrix}$$

It can be verified with a simple exhaustive enumeration that the corresponding CDP matrix is:

$$\text{CDP} = \begin{bmatrix} 1 & 5/1 & 5/1 \\ 1/5 & 1 & 5/5 \\ 1/5 & 5/5 & 1 \end{bmatrix}$$

This is the matrix we assume the decision maker has determined for this example. Clearly, this is the best comparisons a decision maker can make for the fuzzy set of this example.

The formulation (5) that corresponds to this example is:

$$\begin{bmatrix} -1 & 5/1 & 0.0 \\ -1 & 0.0 & 5/5 \\ 0.0 & -1 & 5/5 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} W1 \\ W2 \\ W3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.0 \end{bmatrix}$$

The vector W that minimizes the above Least Squares is calculated to be :

$$W = (0.71429 \;\; 0.14286 \;\; 0.14286)$$

and the corresponding ranking is: $R_1 = 1$, $R_2 = 2$, and $R_3 = 2$.

Obviously, these results contradict the real membership values and ranking of the members of the fuzzy set of this example.

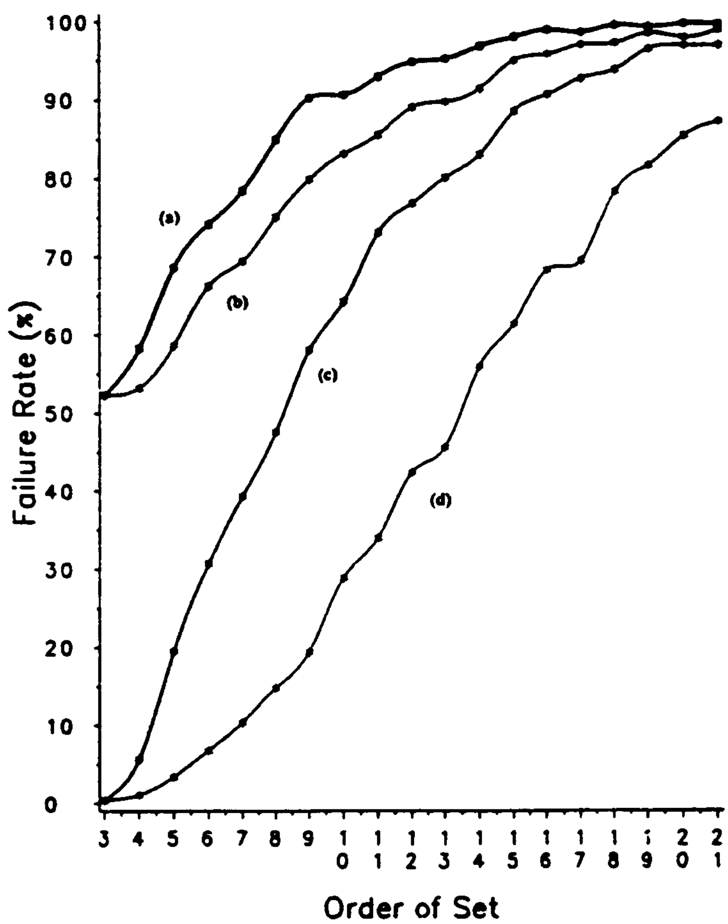

Figure 1.
Failure rates of the Error Minimization and Eigenvalue Method for fuzzy sets of different order
(results are based on one thousand observations).

3. ANALYSIS OF THE FAILURE RATES YIELDED BY THE ERROR MINIMIZATION METHOD

The error minimization method was tested in a similar fashion to the eigenvalue method tested in [28]. More specifically, random problems of different sizes were generated and tested as in the above example. For each such test problem real membership values were generated randomly from the continuous interval (0, 1). However, because the Saaty matrices use values from the set {1/9, 1/8, 1/7, ..., 1, 2, 3, ..., 7, 8, 9} only the random problems that had RCP matrices with entries within the continuous interval [1/9, 9/1] were considered. The RCP matrix, with the real pairwise comparisons was constructed, and after that the CDP matrix was determined.

The error minimization and eigenvalue approaches then were applied. Any contradiction between the real ranking and the ones derived by the error minimization and the eigenvalue methods were recorded as failures. Two kinds of ranking inconsistency were recorded. The first kind is "ranking reversal". For example, if the real ranking of a set of three members is (1, 3, 2) and one method yields (1, 2, 3) then a case of a ranking reversal occurs. The second kind is "ranking indiscrimination". For example, if the real ranking of a set of three members is (1, 3, 2) and one method yields (1, 2, 2) then a case of ranking indiscrimination occurs.

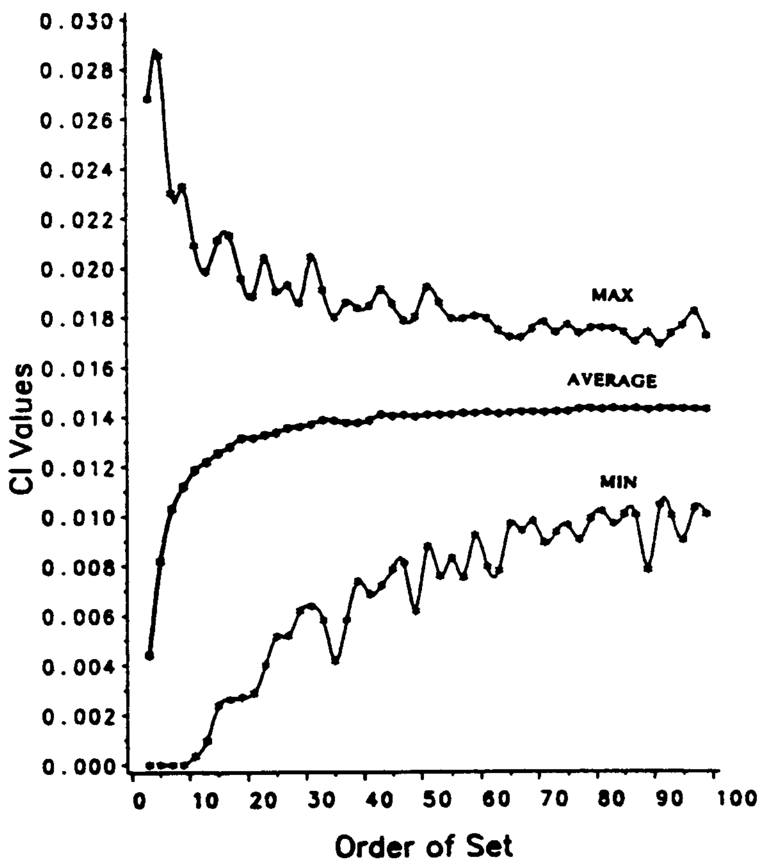

Figure 2.
Maximum, Average, and Minimum CI values of random CDP matrices.

This simulation approach was implemented in FORTRAN with the appropriate IMSL subroutines. Problems of size 3,4,5,...,21 were considered. For each case the number of random matrices generated was 1,000. This number was large enough to ensure that the means converged to within a small error tolerance. The failure rates for fuzzy sets of order: 3, 4, 5,..., 21 are presented in Table 2 and depicted in Figure 1. In Figure 1 curves (a) and (b) represent the total number of failures (adding ranking reversals and ranking indiscriminations) yielded by the eigenvalue and error minimization methods, respectively. Curves (c) and (d) give the number of ranking reversals yielded by the error minimization and eigenvalue methods, respectively. It is important to note here that the CI values of the previously generated CDP matrices are very small [30]. In particular, the CI's are less than 0.029 with an average value equal to 0.0145 (see also Figure 2). These low CI values guarantee that the Dirichlet criterion, as stated by Vargas [31], is satisfied.

Table 2. Failure rates of the Eigenvalue and Error Minimization Method for fuzzy sets of different order (results are based on one thousand observations)

	Eigenvalue Method		Error Minimization Method	
order of set	inversion rate	total failure rate	inversion rate	total failure rate
3	0.4	52.3	0.4	52.3
4	1.1	58.3	5.7	53.2
5	3.4	68.7	19.5	58.7
6	6.8	74.2	30.7	66.3
7	10.4	78.5	39.4	69.5
8	14.8	85.0	47.6	75.1
9	19.4	90.3	58.1	79.9
10	28.9	90.7	64.3	83.2
11	34.0	93.0	73.1	85.6
12	42.4	94.9	76.8	89.1
13	45.6	95.3	80.1	89.8
14	56.0	96.9	83.1	91.5
15	61.4	98.1	88.6	95.1
16	68.3	99.0	90.7	95.9
17	69.5	98.7	92.8	97.1
18	78.3	99.6	93.9	97.3
19	81.7	99.4	96.6	98.6
20	85.5	99.8	97.0	98.0
21	87.3	99.7	97.0	99.0

Although the performance of the error minimization and eigenvalue methods may change if a different set of values for pairwise comparisons is considered, it is interesting to observe that a decision-maker always is restricted to select from a finite set of choices. In 1846 Weber stated his law regarding a stimulus of measurable magnitude. According to his law a change in sensation is noticed if the stimulus is increased by a constant percentage of the stimulus itself [20]. That is, people are unable to make choices from an infinite set. For example, people cannot distinguish between two very close values of importance, say 3.00 and 3.01. Psychological experiments have also shown that individuals cannot simultaneously compare more than seven objects (plus or minus two) [14]. This is the main reason why Saaty uses 9 as the upper limit of his scale, 1 as the lower limit of his scale and a unit difference between successive scale values.

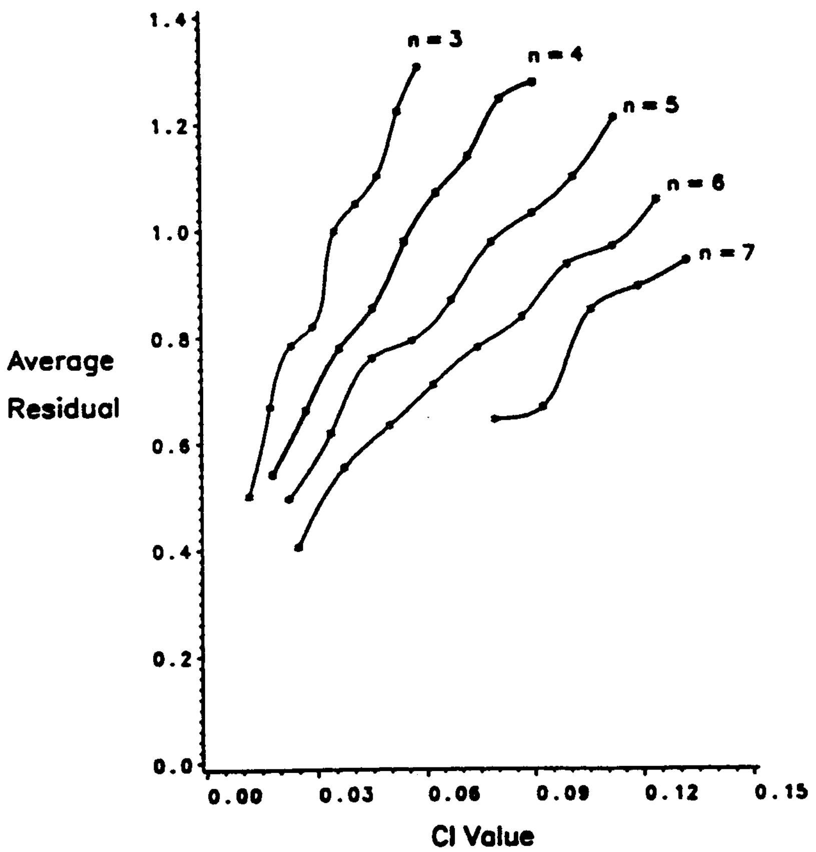

Figure 3.
Average residual and CI versus order of fuzzy set when the Eigenvalue Method is used (results are based on one hundred observations).

The findings in [29] and the present tests reveal that the failure rate for the two methods increases with the number of members in the fuzzy set. Table 2 presents the failure rates of both the eigenvalue and error minimization methods when the above simulation approach was used. The results suggest that the eigenvalue method yields, on the average, more failures than the error minimization approach. However, the number of ranking reversals is significantly higher with the error minimization method. In Triantaphyllou, et. al [28], the two methods were evaluated using a least squared residuals criterion. These results are given in Figures 3 and 4, as they appeared in [28]. Although the error minimization method does better as far as the least squared residuals and the total number of failures is concerned, it does poorly in terms of the number of ranking reversals. Therefore, we cannot conclude that there is a single best method.

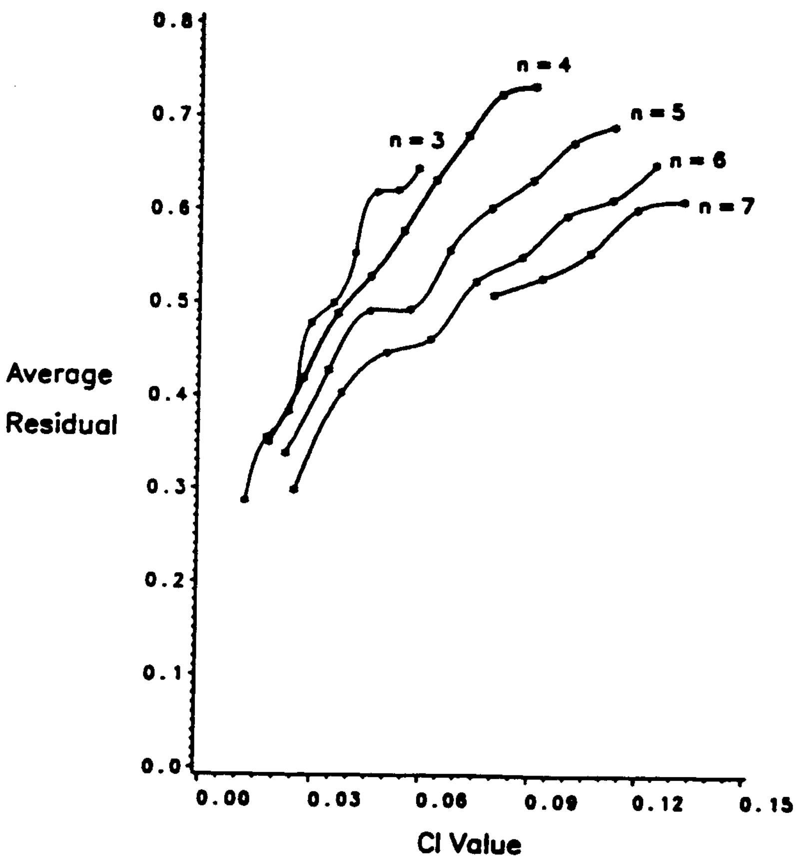

Figure 4.
Average residual and CI versus order of fuzzy set when the Error Minimization Method is used (results are based on one hundred observations).

4. CONCLUDING REMARKS

In this paper we considered various techniques for estimating membership values in fuzzy sets for real world problems when only one decision-maker is involved. The forward error analysis presented here yields a mechanism for testing the effectiveness of methods that evaluate membership values in fuzzy sets when pairwise comparison matrices are used as input data. The same analysis also reveals the magnitude of the problem of correctly evaluating membership values. The error minimization approach, with regard to the total number of failures (Figure 2), seems to be more effective than the eigenvalue approach, though still yielding high failure rates.

In the present paper the assumption is made that the decision-maker deals with a CDP matrix. That is, at any moment the decision-maker is able to determine a value from the set {1/9, 1/8, 1/7, ..., 1,2,3, ..., 7, 8, 9} that is closest to the actual value (which in reality is unknown). The high failure rates derived using the forward error analysis were based on the above assumption which as described previously is biased in favor of the methods. However, under real life conditions the decision-maker may not deal with CDP matrices. This fact suggests that in real life situations the possibility for ranking reversals or ranking indiscriminations is higher than the already high values indicated by the findings of this paper. Since the examined methods use information that comes solely from pairwise comparisons, additional sources of information, such as order information [32], may enhance the power of the pairwise comparison methods. Currently, the high failure rates yielded by either approach in combination with the importance of accurately evaluating membership values make the need for developing more powerful approaches an urgent one.

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