Determining the most important criteria in maintenance decision making

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Introduction

Decision analysis is used when a decision maker wishes to evaluate the performance of a number of alternative solutions for a given problem. Usually, these solutions (also called problem alternatives or just alternatives) can be evaluated in terms of a number of decision criteria. Often these criteria are competing with each other. That is, an alternative may be superior in terms of one or some of the decision criteria, but inferior in terms of some other criteria. The problem is then to identify the best (i.e. the most preferred) alternative and also determine a ranking of the alternatives when all the decision criteria are considered simultaneously. For the above reasons, the branch of decision analysis which deals with this kind of problem is called multi-criteria decision making.

From the previous discussion it becomes evident that multi-criteria decision making (MCDM) is a critical decision tool for many scientific, financial, political, and engineering challenges. The above discussion also indicates the reasons why MCDM problems can be difficult to analyse. Unlike many other decision theories (such as most inventory and scheduling models, linear programming, dynamic programming, etc.), MCDM methodologies are controversial and there is not a unique theory accepted by everyone in the field. The interested reader may want to consult the surveys and analyses reported by Triantaphyllou and Mann[1,2] for an exposure to some of these controversies.

Many of the maintenance challenges can be modelled as MCDM problems. Almeida and Bohoris[3] discussed the application of decision theory to maintenance. According to them, maintenance decisions very often need to consider complex criteria such as repairability, reliability and availability requirements for each one of a set of competing alternative solutions. From the

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last statement it becomes evident that a decision maker may never be sure about the relative importance of decision criteria when dealing with complex maintenance problems. For this reason, an MCDM methodology to be effective needs to be coupled with a sensitivity analysis phase. The main objective of this paper is to present a sensitivity analysis methodology on the decision criteria for a basic MCDM model which has many applications in maintenance decision making.

Multi-criteria decision making – an overview

There are three main steps in utilizing any decision-making technique involving numerical analysis of alternatives:

- Determining the relevant criteria and alternatives.
- Attaching numerical measures to the relative importance (i.e. weights) of the criteria and to the impacts (i.e. the relative performance) of the alternatives on these criteria.
- Processing the numerical values to determine a ranking of each alternative.

Next, consider a decision-making problem with M alternatives and N criteria. In this paper alternatives will be denoted as A_i (for $i = 1, 2, 3, \ldots, M$) and criteria as C_j (for $j = 1, 2, 3, \ldots, N$). We assume that for each criterion C_j the decision maker has determined its importance, or weight, W_j. It is also assumed that the following relationship is always true:

$$\sum_{j=1}^{N} W_j = 1. \tag{1}$$

Furthermore, it is assumed that the decision maker has determined a_{ij} (for $i = 1, 2, 3, \ldots, M$ and $j = 1, 2, 3, \ldots, N$); the measure of performance of alternative A_i in terms of criterion C_j. Some decision methods (for instance, the AHP[4]) require that the a_{ij} values represent *relative* importance. Given the above data and a decision-making method, the objective of the decision maker is to find the best (i.e. the most preferred) alternative.

Let P_i (for $i = 1, 2, 3, \ldots, M$) represent the *final preference* of alternative A_i when all the decision criteria are considered simultaneously. Different decision methods apply different procedures in calculating the values P_i. Without loss of generality, it can be assumed (by a simple rearrangement of the indexes) that the M alternatives are arranged in such a way that the following relation (ranking) is satisfied (that is, the first alternative is always the best alternative and so on):

$$P_1 \geq P_2 \geq P_3 \ldots \geq P_M. \tag{2}$$

The weighted sum model

Probably the simplest and still the widest used MCDM method is the weighted sum model (WSM). The preference P_i of alternative A_i (i = 1, 2, 3, ..., M) is calculated according to the following formula[5]:

$$P_i = \sum_{j=1}^{N} a_{i,j} W_j, \text{for } i = 1,2,3,\ldots,M. \tag{3}$$

Therefore, in the maximization case, the best alternative is the one which corresponds to the largest preference value. The supposition which governs this model is the *additive utility* assumption. However, the WSM should be used only when the decision criteria can be expressed in identical units of measure (e.g. dollars, pounds, seconds).

The analytic hierarchy process

Part of the analytic hierarchy process (AHP)[4,6] deals with the structure of an $M \times N$ matrix A. This matrix is constructed using the *relative importance* of the alternatives in terms of each criterion. The vector ($a_{i,1}$, $a_{i,2}$, $a_{i,3}$, ..., $a_{i,N}$) for each i = 1, 2, 3, ..., M is the principal eigenvector of an $N \times N$ reciprocal matrix which is determined by using pairwise comparisons of the impact of the M alternatives on the ith criterion. Some evidence is presented in [6] which supports the technique for eliciting numerical evaluations of qualitative phenomena from experts and decision makers. For a critical evaluation of the eigenvector approach and the AHP the interested reader is referred to [7-9].

According to AHP the final preference of each alternative is given by the following formula:

$$P_i = \sum_{j=1}^{N} a_{i,j} W_j, \text{for } i = 1,2,3,\ldots,M. \tag{4}$$

Therefore, in the maximization case, the best alternative is the one which corresponds to the highest P_i value.

The similarity between the WSM and the AHP is clear. The AHP uses *relative* values instead of the actual ones. In other words, the following relation is always true in the AHP case:

$$\sum_{i=1}^{N} a_{i,j} = 1, \text{for any } j = 1,2,3,\ldots,N. \tag{5}$$

Thus, it can be used in single or multi-dimensional decision-making problems.

Belton and Gear[10] proposed a revised version of the AHP model. They demonstrated that an inconsistency can occur when the AHP is used. Instead of having the relative values of the alternatives A_1, A_2, A_3, ..., A_M sum up to one (e.g. equation (5) to hold), they propose to divide each relative value by the maximum quantity of the relative values in each column of the matrix A. Later, Saaty[4] accepted the previous notion in a variant of the original AHP called the *ideal mode* AHP. Besides the previous two MCDM methods more methods are available (for a survey see, for instance, [11]).

Quantifying data by using pairwise comparisons

One of the most crucial steps in many decision-making methods, including the ones related to maintenance management, is the accurate estimation of the pertinent data. This is a problem not bound in the AHP method only, but it is crucial in many other methods which need to elicit qualitative information from the decision maker. Very often qualitative data cannot be known in terms of absolute values. For instance, "what is the worth of a specific computer software in terms of a user adaptivity criterion?" Although information about questions like the previous one is vital in making the correct decision, it is very difficult, if not impossible, to quantify them correctly. Therefore, many decision-making methods attempt to determine the *relative* importance, or weight, of the alternatives in terms of each criterion involved in a given decision-making problem.

An approach based on pairwise comparisons which was proposed by Saaty[6] has long attracted the interest of many researchers. Pairwise comparisons are used to determine the relative importance of each alternative in terms of each criterion. In this approach the decision maker has to express his/her opinion about the value of one single pairwise comparison at a time. Usually, the decision maker has to choose his/her answer among 10-17 discrete choices (see also Table I).

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience and judgement slightly favour one activity over another
5	Essential or strong importance	Experience and judgement strongly favour one activity over another
7	Demonstrated importance	An activity is strongly favoured and its dominance demonstrated in practice
9	Absolute importance	The evidence favouring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values between the two adjacent judgements	When compromise is needed
Reciprocals of above non-zero	If activity *i* has one of the above non-zero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*	

Table I. Scale of relative importances (according to Saaty[6])

A central problem with the pairwise comparisons is how to quantify the linguistic choices selected by the decision maker during their evaluation. All the methods which use the pairwise comparisons approach eventually express the qualitative answers of a decision maker into some numbers which, most of the time, are ratios of integers. A case in which pairwise comparisons are expressed as differences (instead of ratios) was used to define similarity relations and is described by Triantaphyllou[12]. The following paragraphs examine the issue of quantifying pairwise comparisons. Since pairwise comparisons are the keystone of these decision-making processes, correctly quantifying them is the most crucial step in multi-criteria decision-making methods which use qualitative data.

Pairwise comparisons are quantified by using a scale. Such a scale is a one-to-one mapping between the set of discrete linguistic choices available to the decision maker and a discrete set of numbers which represent the importance, or weight, of the previous linguistic choices. The scale proposed by Saaty is depicted in Table I. Other scales have also been proposed by others. An evaluation of 78 different scales appears in Triantaphyllou *et al.*[13]. All alternative scales depart from some psychological theories and develop the numbers to be used based on these psychological theories.

The values of the pairwise comparisons in the AHP are determined according to the scale introduced by Saaty[6]. According to this scale, the available values for the pairwise comparisons are members of the discrete set: $\{9, 8, 7, 6, 5, 4, 3, 2, 1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7, 1/8, 1/9\}$ (see also Table I).

As an illustrative example consider the following situation. Suppose that in selecting the best computer system, there are three alternative configurations, say A, B, and C. Also, suppose that one of the decision criteria is hardware expandability (i.e. the flexibility of attaching to the system other related peripheral devices, such as printers, new memory, etc.). Suppose that system A is much better than system B, and system C is the least desirable one as far as the hardware expandability criterion is concerned. Also, suppose that Table II is the judgement matrix when the three alternative configurations are examined in terms of this criterion.

For instance, when system A is compared to system B, the decision maker has determined that system A is between being classified as "essentially more important" and "demonstrated more important" than system B (see also Table I). Thus, the corresponding comparison assumes the value of 6 (entry (1, 2) in the

Table II.
Judgement matrix

C_1 Hardware expandability	A	B	C
A	1	6	8
B	1/6	1	4
C	1/8	1/4	1

previous judgement matrix. A similar interpretation is true for the rest of the entries.

The next step is to extract the relative importances implied by the previous comparisons. That is, how important are the three alternatives when they are considered in terms of the hardware expandability criterion? Saaty asserts that to answer this question one has to estimate the right principal eigenvector of the previous matrix. Given a judgement matrix with pairwise comparisons, the corresponding maximum left eigenvector is approximated by using the geometric mean of each row. That is, the elements in each row are multiplied with each other and then the nth root is taken (where n is the number of elements in the row). Next the numbers are normalized by dividing them with their sum. Hence, for the previous matrix the corresponding priority vector is: (0.754, 0.181, 0.065).

An evaluation of the eigenvalue approach can be found in [9]. An alternative approach for evaluating the relative priorities from a judgement matrix is based on a least squares formulation and is described in [8]. One of the most practical issues in the AHP methodology is that it allows for slightly inconsistent pairwise comparisons. If all the comparisons are perfectly consistent, then the following relation: $a_{ij} = a_{ik}\, a_{kj}$ should always be true for any combination of comparisons taken from the judgement matrix.

However, perfect consistency rarely occurs in practice. In the AHP the pairwise comparisons in a judgement matrix are considered to be adequately consistent if the corresponding consistency ratio (CR) is less than 10 per cent[6]. The CR coefficient is calculated as follows. First the consistency index (CI) needs to be estimated. This is done by adding the columns in the judgement matrix and multiplying the resulting vector by the vector of priorities (i.e. the approximated eigenvector) obtained earlier. This yields an approximation of the maximum eigenvalue, denoted by λ_{max}. Then, the CI value is calculated by using the formula: $CI = (\lambda_{max} - n)/(n - 1)$.

The concept of the RCI (random consistency index) is used next. Given a value of n (e.g. the number of items to be compared) the RCI value corresponds to the average random consistency index (calculated by using the formula $CI = (\lambda_{max} - n)/(n - 1)$). The concept of the RCI was also introduced by Saaty in order to establish (by means of a statistical test of hypothesis) an upper limit on how much inconsistency may be tolerated in a decision process. The RCI values for different n values are given in Table III. Next, the CR value of a judgement matrix is obtained by dividing the CI value by the corresponding RCI value as given in Table III. If the CR value is greater than 0.10, then a re-evaluation of the pairwise comparisons is recommended (because the corresponding consistency

n	1	2	3	4	5	6	7	8	9
RCI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

Table III. RCI values for different values of n

ratio is considered as high). This is repeated until a CR value of 0.10 or less is achieved.

When the above approximations are applied to the previous judgement matrix, it can be easily shown that the following are derived: $\lambda_{max} = 3.131$, CI = 0.068, and CR = 0.117. For instance, $\lambda_{max} = 3.131$ because $(1 + 1/6 + 1/8) \times (0.754) + (6 + 1 + 1/4) \times (0.181) + (8 + 4 + 1) \times (0.065) = 3.131$. Recall that if the value of CR is greater than 0.10, then it is a good idea to study the problem further and re-evaluate the pairwise comparisons (this was not done in this numerical example due to the brevity of the presentation).

After the alternatives are compared with each other in terms of each one of the decision criteria and the individual priority vectors are derived, the synthesis step is taken. The priority vectors become the columns of the decision matrix (not to be confused with the judgement matrices with the pairwise comparisons). The weights of importance of the criteria are also determined by using pairwise comparisons. Therefore, if a problem has M alternatives and N criteria, then the decision maker is required to construct N judgement matrices (one for each criterion) of order $M \times M$ and one judgement matrix of order $N \times N$ (for the N criteria). Finally, given a decision matrix the final priorities, denoted by P_i, of the alternatives in terms of all the criteria combined are determined according to the previous formula (4).

Determining the most critical criterion

In this paper we determine which is the most critical decision criterion. In the next paragraphs we follow the discussion and developments proposed in [14].

Intuitively, one may think that the most critical criterion is the criterion which has the highest weight W_j[15]. However, this notion of criticality may be misleading. In the proposed methodology the most critical criterion is the one for which the smallest change in its current weight will alter the existing ranking of the alternatives. In the previous statement the concept "smallest change" will be defined in relative terms (i.e., as a percentage of change as opposed to absolute terms). Next, we introduce the required definitions and notation used to deal with this problem.

Definition 1: Let $\delta_{k,i,j}$ ($1 \leq i < j \leq M$ and $1 \leq k \leq N$) denote the minimum change, in absolute terms, in the current weight W_k of criterion C_k such that the ranking of alternatives A_i and A_j will be reversed.

At this point a related issue is how the weights of the decision criteria can be reallocated, after a change $\delta_{k,i,j}$ is considered. For an easy illustration suppose that a change $\delta_{1,1,2}$ is considered for the weight of the first criterion (any other change can be dealt with in a similar manner). That is, in the current setting we have: $k = 1$, $i = 1$ and $j = 2$. Therefore, the new (i.e. modified) weight, denoted as W_1^*, of the first criterion is:

$$W_1^* = W_1 - \delta_{1,1,2}.$$

In order to preserve property (1), it is necessary that all weights be normalized. Therefore, the new normalized weights, denoted as W_i', will be as follows:

$$W_1' = \frac{W_1^*}{W_1^* + W_2 + \ldots + W_n}$$
$$W_2' = \frac{W_2}{W_1^* + W_2 + \ldots + W_n}$$
$$\vdots$$
$$W_n' = \frac{W_n}{W_1^* + W_2 + \ldots + W_n}. \tag{6}$$

Next, define: $\delta'_{k,i,j} = \delta_{k,i,j} \times 100 / W_k$, for any $1 \le i < j \le M$ and $1 \le k \le N$. That is, $\delta'_{k,i,j}$ expresses changes in relative terms (i.e. in percentage). The above quantities can be calculated according to formula (7a) of Theorem 1 (presented later).

Then, the most critical criterion is defined in two possible ways (recall that from relation (2) alternative A_1 is always assumed to be the best alternative). The first definition applies when one is interested only in changes in the best (top) alternative, while the second definition applies when one is interested in changes in the ranking of any alternative. Recall that $|s|$ stands for the absolute value function (e.g. $|-5| = +5$, etc.).

Definition 2: The per cent-top (or PT) critical criterion is the criterion which corresponds to the smallest $|\delta'_{k,i,j}|$ $(1 \le j \le M$ and $1 \le k \le N)$ value.

Definition 3: The per cent-any (or PA) critical criterion is the criterion which corresponds to the smallest $|\delta'_{k,i,j}|$ $(1 \le i < j \le M$ and $1 \le k \le N)$ value.

Besides the previous definitions, which are based on relative term changes, one may wish to consider changes in absolute terms and thus derive two additional definitions. However, that might become overwhelming to the decision maker. After all, a change, say of 0.03, does not mean much unless someone is also given the original value. A change of 0.03 is very different if the original value was 0.08 or 0.80. That is, it is more meaningful to use relative changes. Therefore, in this paper the emphasis will be on relative (percentage) changes and thus all developments are based on relative changes. The following two definitions express how critical a given decision criterion is.

Definition 4: The criticality degree of criterion C_j, denoted as D_j', is the smallest amount (percentage) by which the current value of W_j must change, such that the existing ranking of the alternatives will change. That is, the following condition is true:

$$D'_k = \min_{1 \le i < j \le M} \left\{ \left| \delta'_{k,i,j} \right| \right\}, \text{for all } N \ge k \ge 1.$$

Definition 5: The sensitivity coefficient of criterion C_j, denoted as sens(C_j), is the reciprocal of its criticality degree. That is, the following condition is true:

$$sens(C_j) = \frac{1}{D'_j}, \text{for any } N \ge j \ge 1.$$

If the criticality degree is infeasible, then the sensitivity coefficient is set equal to zero.

In [14] a theorem is given for the AHP (original and ideal mode) and WSM methods and it is described as follows (recall that currently the following relation is assumed to be true from (2): $P_i \ge P_j$, for all $1 \le i \le j \le M$):

Theorem 1: When the WSM, AHP, or ideal mode AHP methods are used, the quantity $\delta'_{k,i,j}$ ($1 \le i < j \le M$ and $1 \le k \le N$), by which the current weight W_k of criterion C_k needs to be modified (after normalization) so that the ranking of the alternatives A_i and A_j will be reversed, is given as follows:

$$\begin{aligned} \delta'_{k,i,j} &\le \frac{(P_j - P_i)}{(a_{jk} - a_{ik})} \times \frac{100}{W_k}, \text{if } (a_{jk} > a_{ik}) \text{ or :} \\ \delta'_{k,i,j} &\ge \frac{(P_j - P_i)}{(a_{jk} - a_{ik})} \times \frac{100}{W_k}, \text{if } (a_{jk} < a_{ik}). \end{aligned} \tag{7a}$$

Furthermore, the following condition should also be satisfied for the value to be feasible:

$$\frac{(P_j - P_i)}{(a_{jk} - a_{ik})} \times \frac{100}{W_k} \le 100. \tag{7b}$$

Given the above formulas, one can see that the value of $\delta'_{k,i,j}$ should be equal to:

$$\frac{(P_j - P_i)}{(a_{jk} - a_{ik})} \times \frac{100}{W_k}.$$

Also, from the previous considerations it can be seen that if alternative A_i dominates alternative A_j (i.e. $a_{ik} \ge a_{jk}$, for any $k = 1, 2, 3, \ldots, N$) then, it is impossible to make alternative A_j more preferred than alternative A_i by changing the weights of the criteria. Also, a criterion C_k is a robust criterion if all the $\delta'_{k,i,j}$ ($1 \le i < j \le M$ and $1 \le k \le N$) quantities associated with it are infeasible. In other words, if the following condition is true:

$$\frac{(P_j - P_i)}{(a_{jk} - a_{ik})} \times \frac{100}{W_k} > 100, \text{for all } i, j = 1, 2, 3, \ldots, M$$

for some criterion C_k then, any change on the weight of that criterion does not affect the existing ranking of any of the alternatives and thus this criterion is a robust one.

Therefore, if one is interested in determining the most critical criterion, then all possible $\delta'_{k,i,j}$ ($1 \leq i < j \leq M$ and $1 \leq k \leq N$) values need to be calculated. Observe that there are $N \times (M(M-1))/2$ such $\delta'_{k,i,j}$ quantities.

An illustrative example

The numerical data in this example have been adapted from an example in [14]. Suppose that a decision analyst is considering four alternative systems, say A_1, A_2, A_3, and A_4. Each system has been evaluated in terms of four maintenance related criteria. Suppose that these four criteria are as follows: C_1 = cost, C_2 = repairability, C_3 = reliability, and C_4 = availability. Since these criteria cannot be expressed in terms of the same unit of measurement, the WSM model cannot be applied. Therefore, the decision analyst has to express the data in relative terms (i.e. to apply the AHP method). That is, the decision maker has to express the relative importance of the four alternatives when they are examined in terms of a single decision criterion at a time.

Moreover, suppose that the decision analyst has determined that the relative weights of importance of the four criteria are as follows: W_1 = 0.3277, W_2 = 0.3058, W_3 = 0.2876, and W_4 = 0.0790. The columns of the decision matrix and weights of importance of the four decision criteria, can be determined by using sequences of pairwise comparisons, as it was demonstrated in the illustrative example in the earlier section which discussed the pairwise comparison concept. Suppose that Table IV is the decision matrix for this problem. For instance, it is assumed that when the cost criterion is considered, then alternative A_1 is (0.3088/0.2163) times more preferred when it is compared to alternative A_2. Also observe that all the data are expressed in relative terms. For a more detailed description of how relative data can be derived, the interested reader may want to consult the work in [4] or [13]. Next, the AHP approach is applied (that is, formula (2) is used) and the final preferences and ranking of the four alternatives are as shown in Table V.

Alternative	Maintenance criteria			
	C_1: cost	C_2: repairability	C_3: reliability	C_4: availability
A_1	0.3088	0.2897	0.3867	0.1922
A_2	0.2163	0.3458	0.1755	0.6288
A_3	0.4509	0.2473	0.1194	0.0575
A_4	0.0240	0.1172	0.3184	0.1215

Table IV. Decision matrix example

Therefore, the relation $P_1 \geq P_2 \geq P_3 \geq P_4$ holds and as result the most preferred alternative system is A_1. Next, observe that according to the weights of the four criteria, criterion C_1 (i.e. cost) appears to be the most important one. The minimum quantity $\delta_{1,1,3}$ (i.e., when measured in absolute terms) needed to alter the current weight W_1, so that the current ranking of the two alternatives A_1 and A_3 will be reversed, can be found by using relation (7a) of Theorem 1 and definition 1 as follows:

$$\delta_{1,1,3} \leq \frac{(0.2621 - 0.3162)}{(0.4509 - 0.3088)} = -0.3807.$$

Alternative	Preference (P_j)	Ranking
A_1	0.3162	1[a]
A_2	0.2768	2
A_3	0.2621	3
A_4	0.1449	4

Note: [a]Indicates the most preferred (i.e. best) alternative

Table V.
Current final preferences

Therefore, the value of $\delta_{1,1,3}$ must be equal to –0.3807 and consequently the value of $\delta'_{1,1,3}$ is equal to –116.1733 (= –0.3807 × (100/0.3277)). This $\delta'_{1,1,3}$ value is feasible since from (7b) it is less than 100. Thus, the modified weight W_1^* of the first criterion (before normalization) for this case is:

$$W_1^* = [\,0.3277 - (-0.3807)] = 0.7084.$$

One may want to observe at this point that when relations (6) are applied on the previous data, then the reallocated criteria weights become: $W_1' = 0.5130$, $W_2' = 0.2215$, $W_3' = 0.2083$, and $W_4' = 0.0572$. It can easily be shown that now the new final preferences are: $P_1' = 0.3141$, $P_2' = 0.2601$, $P_3' = 0.3142$, and $P_4' = 0.1115$. That is, now the alternatives are ranked as follows: $A_3 > A_1 > A_2 > A_4$ (where ">" stands for "better than").

Working as above for all the possible combinations of criteria and pairs of alternatives, Table VI is derived. Table VII depicts the changes in relative terms (that is, the $\delta'_{k,i,j}$ values which are calculated from the relation $\delta'_{k,i,j} = \delta'_{k,i,j} \times 100 / W_k$). Observe that negative changes indicate increases, while positive changes indicate decreases.

The percentage-top (PT) critical criterion can be found by looking for the smallest relative value (in absolute value terms) of all the rows which are related to alternative A_1 (i.e. the best alternative) in Table VII. The smallest such percentage (64.8818 per cent) corresponds to criterion C_3 (reliability) when the pair of alternatives A_1 and A_2 is considered. For criterion C_3 a reduction of the current weight by 64.8818 per cent will make A_2 the most preferred alternative and A_1 will not be the best alternative any more.

The percentage-any (PA) critical criterion can be found by looking for the smallest relative $\delta'_{k,i,j}$ value in Table VII. Such smallest value is $\delta'_{3,2,3} = 9.1099$ per cent and it (again) corresponds to criterion C_3. Therefore, the PA critical criterion is C_3. Finally, observe that it is a coincidence that both definitions of the most critical criterion indicate the same criterion (i.e. criterion C_3) in this example.

When definition 4 is used, then from Table VII it follows that the criticality degrees of the four criteria are: $D'_1 = |-19.1334| = 19.1334$, $D'_2 = 48.7901$, $D'_3 = 9.1099$, and $D'_4 = 32.5317$. Therefore, the sensitivity coefficients of the decision criteria (according to definition 5) are: sens(C_1) = 0.0523, sens(C_2) = 0.0205, sens(C_3) = 0.1098, and sens(C_4) = 0.0307. That is, the most sensitive decision criterion is C_3 (reliability), followed by criteria C_1, C_4, and C_2.

Concluding remarks

The previous discussion and illustrative example provide a methodology and the reasons why a sensitivity analysis is a necessity when dealing with complex maintenance multi-criteria decision-making problems. The inherent difficulties of accurately assessing the importance of the various maintenance criteria make the application of the previous sensitivity analysis an integral part of the solution process. The results derived by using the proposed methodology can enhance and improve the understanding of the dynamics of a complex

Pair of alternatives	Criterion C_1	C_2	C_3	C_4
A_1-A_2	N/F	–0.7023	0.1866	–0.0902
A_1-A_3	–0.3807	N/F	0.2024	N/F
A_1-A_4	N/F	N/F	N/F	N/F
A_2-A_3	–0.0627	0.1492	0.0262	0.0257
A_2-A_4	N/F	N/F	–0.9230	N/F
A_3-A_4	0.2745	N/F	–0.5890	–1.8313

Note: N/F, non-feasible; that is, the corresponding δ value does not satisfy relation (7b)

Table VI. All possible $\delta_{k,i,j}$ values (in absolute terms)

Pair of alternatives	Criterion C_1	C_2	C_3	C_4
A_1-A_2	N/F	–229.7	**–64.8818**	–114.1772
A_1-A_3	–116.1733	N/F	70.3755	N/F
A_1-A_4	N/F	N/F	N/F	N/F
A_2-A_3	–19.1334	48.7901	**9.1099**	32.5317
A_2-A_4	N/F	N/F	–320.9	N/F
A_3-A_4	83.7656	N/F	–204.8	–2318.10

Note: N/F, non-feasible

Table VII. All possible $\delta'_{k,i,j}$ values (in relative terms) maintenance problem. All experts agree that the better the understanding of the problem, the more successful a solution can be. The proposed methodology provides some of the means for achieving this goal for maintenance problem solving.

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