Neural Network Weight Selection Using Genetic Algorithms

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Neural Networks

Neural networks generally consist of five components.

- A directed graph known as the network topology whose nodes represent the neurodes (or processing elements) and whose arcs represent the connections.
- A state variable associated with each neurode.
- A real-valued weight associated with each connection.
- A real-valued bias associated with each neurode.
- A transfer function f for each neurode such that the state of the neurode is $f(\omega_i x_i - \beta)$.

Genetic Algorithms

Genetic algorithms require five components.

- A way of encoding solutions to the problem on chromosomes.
- An evaluation function which returns a rating for each chromosome given to it.
- A way of initializing the population of chromosomes.
- Operators that may be applied to parents when they reproduce to alter their genetic composition. Standard operators are mutation and crossover.
- Parameter settings for the algorithm, the operators, and so forth.

Genetic Algorithms

$$(19.3, 0.05, -1.2, 345, 2.0, 7.7, 68.0)$$

$$\xrightarrow{Mutation} (19.3, 0.05, -1.2, 345, 2.0, \mathbf{8.2}, 68.0)$$

Figure 1: Mutation Operation

$$(19.3, 0.05, -1.2, 345, 2.0, 7.7, 68.0), (17.6, 0.05, -1.2, 250, 3.0, 7.7, 70.0)$$

$$\xrightarrow{Crossover} (17.6, 0.05, -1.2, 345, 3.0, 7.7, 68.0)$$

Figure 2: Crossover Operation

Genetic Algorithms

Given these five components, a genetic algorithm operates according to the following steps.

- Initialize the population using the initialization procedure, and evaluate each member of the initial population.
- Reproduce until a stopping criterion is met.

Genetic Algorithms

Reproduction consists of iterations of the following steps.

- Choose one or more parents to reproduce. Selection is stochastic, but the individuals with the highest evaluations are favored in the selection.
- Choose a genetic operator and apply it to the parents.
- Evaluate the children and accumulate them into a generation. After accumulating enough individuals, insert them into the population, replacing the worst current members of the population.

Pattern Classification Problem

A Problem with k features and M classes:

Given a set of training examples, selects the most likely class for any instance not in the training set.

An **instance** is represented by a k-dimensional feature vector.

An **example** is represented by a instance and the class that the instance belongs to.

Real Examples

- Handwritten character recognition
- Speech recognition
- Blood cell classification
- $\ldots$

Neural Networks

- Sigmoid Feed-forward Neural Network
- Weighted Probabilistic Neural Networks (WPNN)

Sigmoid Feed-forward Networks

A feed-forward network

- A directed graph without cycles.
- A multi-layer network
  Neurones in each layer (except output layer) are completely connected to the forward layer.
- Each neuron in the network is a sigmoid unit

Sigmoid Network Topology

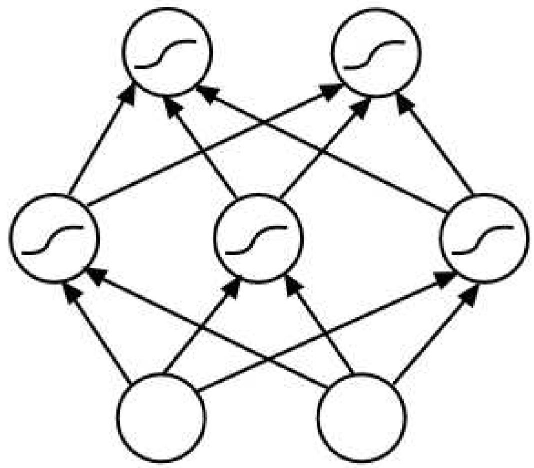
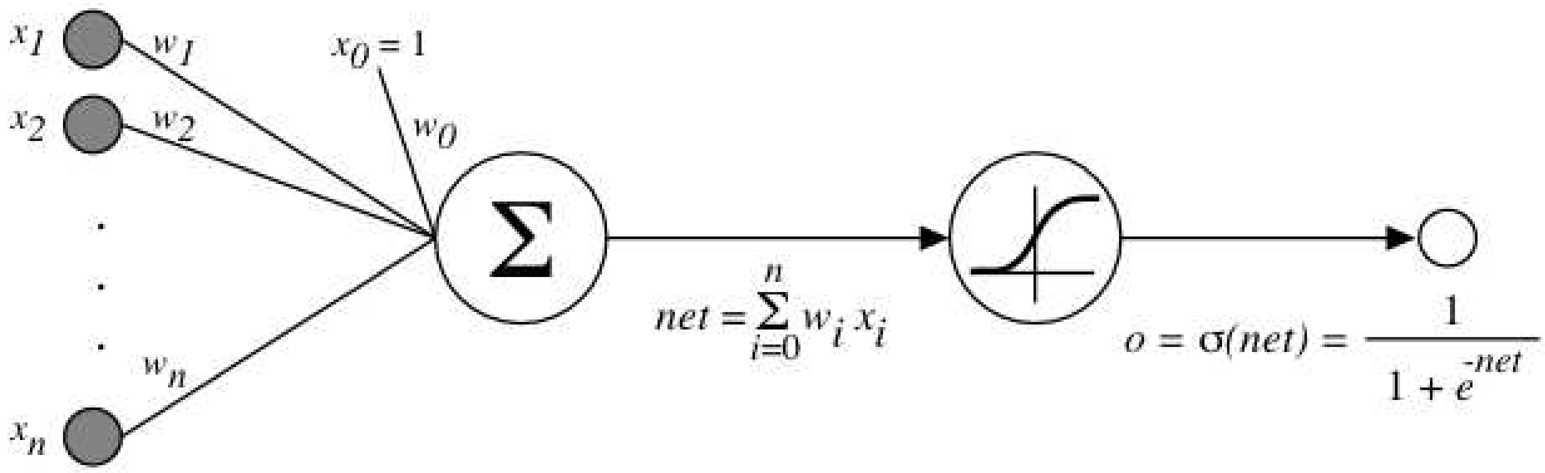


Figure 3: Typical Sigmoid Network

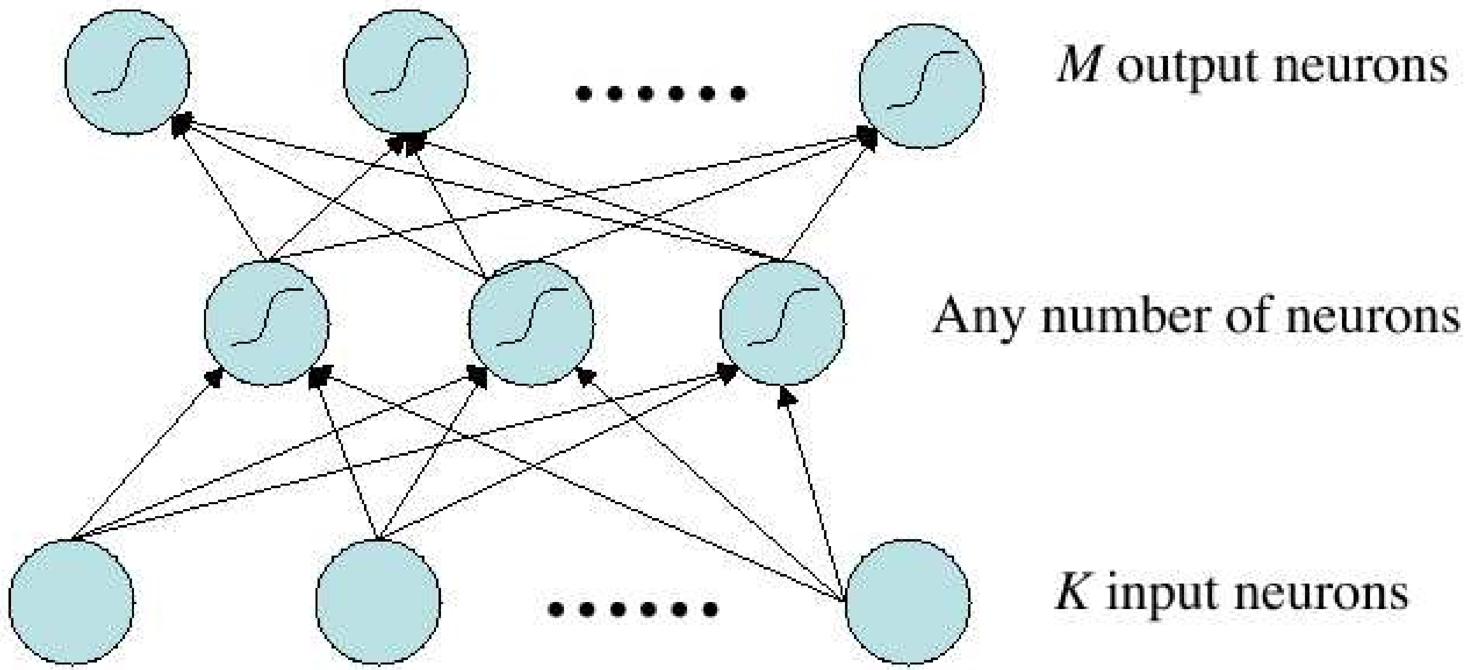
Sigmoid Unit



$\sigma(x)$ is the function:

$$\frac{1}{1+e^{-x}}$$

Application for Pattern Classification



Application for Pattern Classification (Cont.)

- Representation:
  - Input representation: $< x_1, x_2, \ldots, x_k >$
  - Class representation: $< 1, 0, \ldots, 0 >$ for class 1
- Evaluation Function

  The sum of squared errors
- Training Algorithms
  - Backpropagation Algorithm
  - Genetic Algorithm

Weighted Probabilistic Neural Network (WPNN)

- WPNN is a pattern classification algorithm which falls into the broad class of ”nearest-neighbor-like” algorithms.
- Likelihood Function: “distance” → probability

Note: Distance is between an instance to all examples of a particular class, not just one example.

WPNN Likelihood Function

"Let the exemplars from class i be the k-vector $\vec{x}_j^i$ for $j = 1, \ldots, N_i$. Then the likelihood function for class i is

$$L_i(\vec{x}) = \frac{1}{N_i (2\pi)^{k/2} (det\Sigma)^{1/2}} \sum_{j=1}^{N_i} e^{-(\vec{x}-\vec{x}_j^i)^T \Sigma^{-1} (\vec{x}-\vec{x}_j^i)}$$

$L_i(\vec{x})$ describes the probability that the instance x is of class i."

Note: the class likelihood functions are sums of identical anisotropic Gaussians centered at the examples divided by N_i.

WPNN Conditional Probability

The value of the likelihood function for a particular instance may fail to classify the instance. For example, $L_1(\vec{x}) = 0.3, L_j(\vec{x}) = 0.01$ for $j = 2, 3, \ldots, M$.

The **conditional probability** for class i:

$$P_i(\vec{x}) = L_i(\vec{x}) / \sum_{j=1}^{M} L_j(\vec{x})$$

WPNN Feature Weights

- Training WPNN consists of selecting the entries of matrix Σ.
- Matrix Σ is restricted to be a diagonal matrix.
- The inverse of each entry in Σ is a weight on the corresponding feature.

WPNN Implementation

It is called a ”neural network” because its implementation is natural mapped onto a two-layer feed-forward network:

- k neurons in the input layer
- M neurons with standard normal Gaussian transfer functions in the output layer
- kN neurons in the single hidden layers, each with a linear function $ax - b$.

k: number of features, M: number of classes N: number of total examples

WPNN Implementation (Cont.)

The $((i-1)k+j)th$ hidden neuron $(i < N, j < k)$:

1. is connected to the jth input neuron with weight $a = 1.0$
2. has bias b equal to the value of jth feature of the ith example
3. is connected to the nth output neuron, where n is the class of the ith exemplar, with weight w_j , where w_j is the selected feature weight for the jth feature.

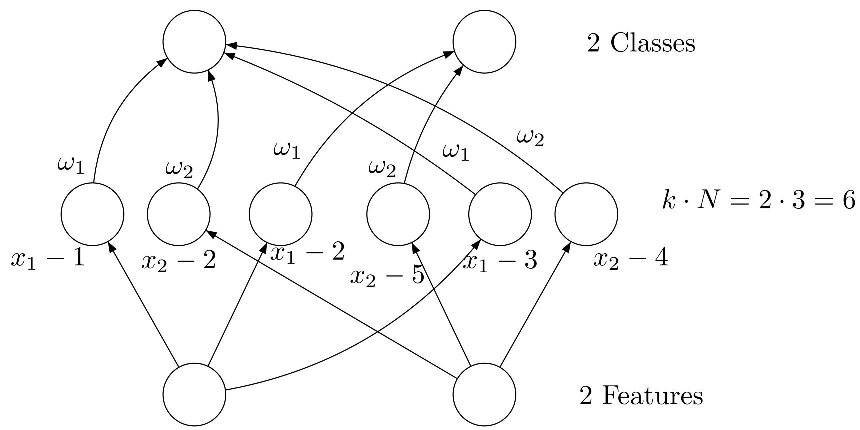
An Example

Two classes, two features, and three examples:

No.	Instance	Class
1	$< 1, 2 >$	1
2	$< 2, 5 >$	2
3	$< 3, 4 >$	1

An Example (Cont.)

$$\frac{1}{N_i (2\pi)^{2k} \frac{1}{\sqrt{\omega_1, \ldots, \omega_k}}} \left(\sum_{j=1}^{N_i} e^{-\sum_{n=1}^{k} \omega_n x_{jn}^2} \right)$$



Survey of Hybrid Systems

- Supportive combinations
  Supportive combinations typically involve using one of these methods to prepare data for consumption by the other. For example, using a genetic algorithm to select features for use by neural network classifiers.

- Collaborative combinations
  Collaborative combinations typically involve using the genetic algorithm to determine the neural network weights or the topology or learning algorithm.

Supportive Combinations

- Using NN to assist GA
- Using GA to assist NN
  - Data preparation
  - Evolving network parameters and learning rules
  - Using GA to explain and analyze NN
- Using GA and NN independently

Collaborative Combinations

GA to select weight
Two basic differences between different approaches: architectures (feedforward sigmoidal, WPNN, cascade-correlation, recurrent sigmoidal, recurrent linear threshold, feedforward with step functions and feedback with step functions) and difference in GA itself.

Collaborative Combinations

- GA to specify NN topology
  - A genotype representation must be devised and an attendant mapping from genotype to phenotype must be provided.
  - There must be a protocol for exposing the phenotype to the task environment.
  - There must be a learning method to fine tune the network function.
  - There must be a fitness measure.
  - There must be method for generating new genotypes.
- GA to learn the NN learning algorithm

Reasons to apply GA to NN

- Finding the global optima
- For recurrent networks
- For networks with discontinuous functions
- GA can optimize any combination of weights, biases, topology and transfer functions
- The ability to use arbitrary evaluation function

Disadvantage and Solutions

- Excessive computing time
- Solutions:
  - Using specialized NN hardware
  - Using the best local learning algorithm
  - Using parallel implementations of GA
  - By finding the best division of labor between the local and evolutionary learning paradigms to make the best possible use of training time

GA Formulation for Training Sigmoid FFNN

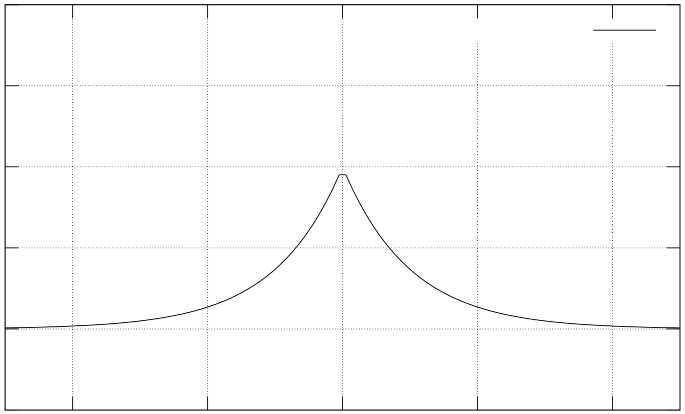
- Problem Formulation
- Initialization
- Genetic Operators
- Fitness Evaluation
- Genetic Parameters

Problem Formulation

- Individuals are the NN's themselves;
- no string encoding
- Topology is fixed, weights are evolved

Population Initialization

- Selection of initial weights



Plot of $e^{-|x|}$

Fitness Evaluation

- Sum of Squared Error

$$E(\vec{\omega}) \equiv 1/2 \sum_{d \in D} \sum_{k \in outputs} (t_{kd} - o_{kd})^2$$

Genetic Operators - Sigmoid FFNN

- Unbiased-Mutate-Weights - Select from probability distribution
- Biased-Mutate-Weights - Add in previous weight
- Mutate-Nodes - Mutatation grouped by node (schema preservation)
- Crossover-Weights - Uniform-crossover vs. point-crossover
- Crossover-Nodes - Crossover grouped by node (schema preservation)
- Crossover-Features - Competing conventions
- Mutate-Weakest-Nodes - Non-random selection of least contributive
- Hillclimb - One step in direction of gradient

Parameter Values

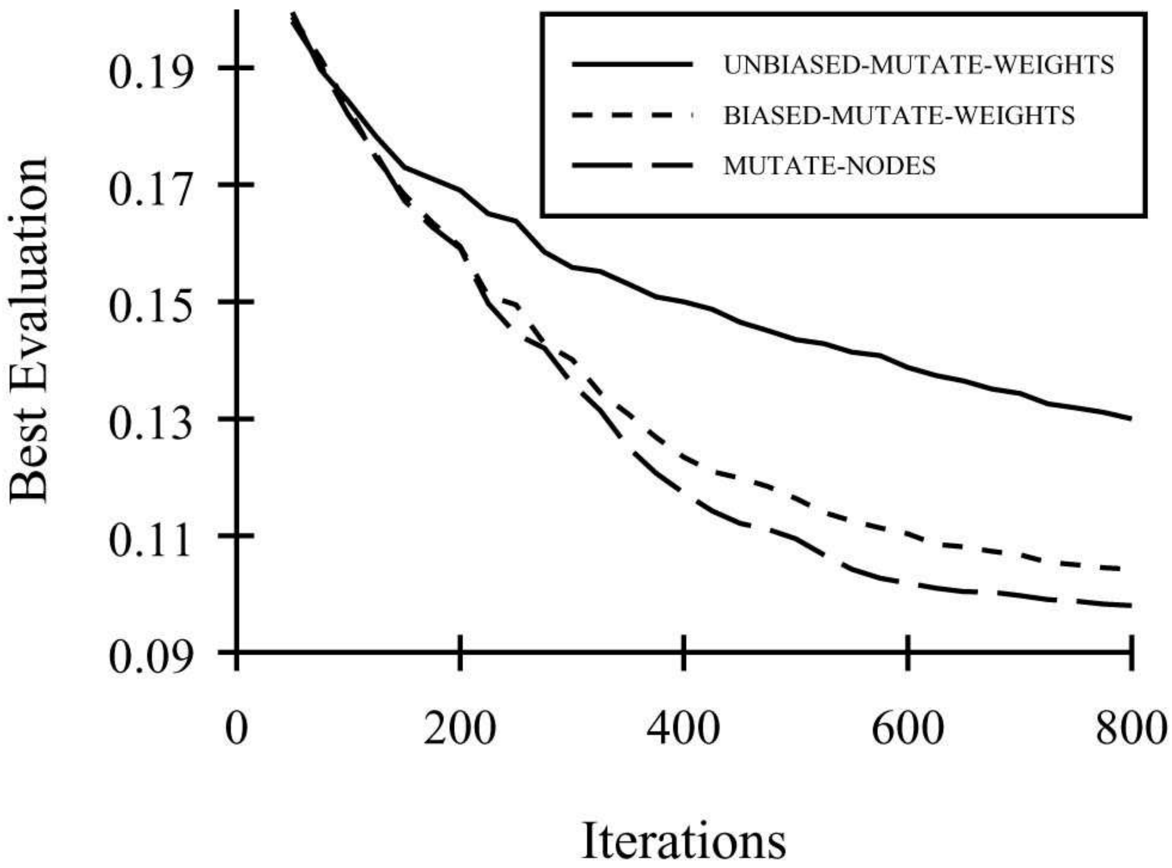
- Population-Size: 50
- Generation-Size: 1
- Parent-Scalar: [0.89, 0.93]

Experimental Results — Feedforward Sigmoidal

First, Comparing the three mutations resulted in a clear order

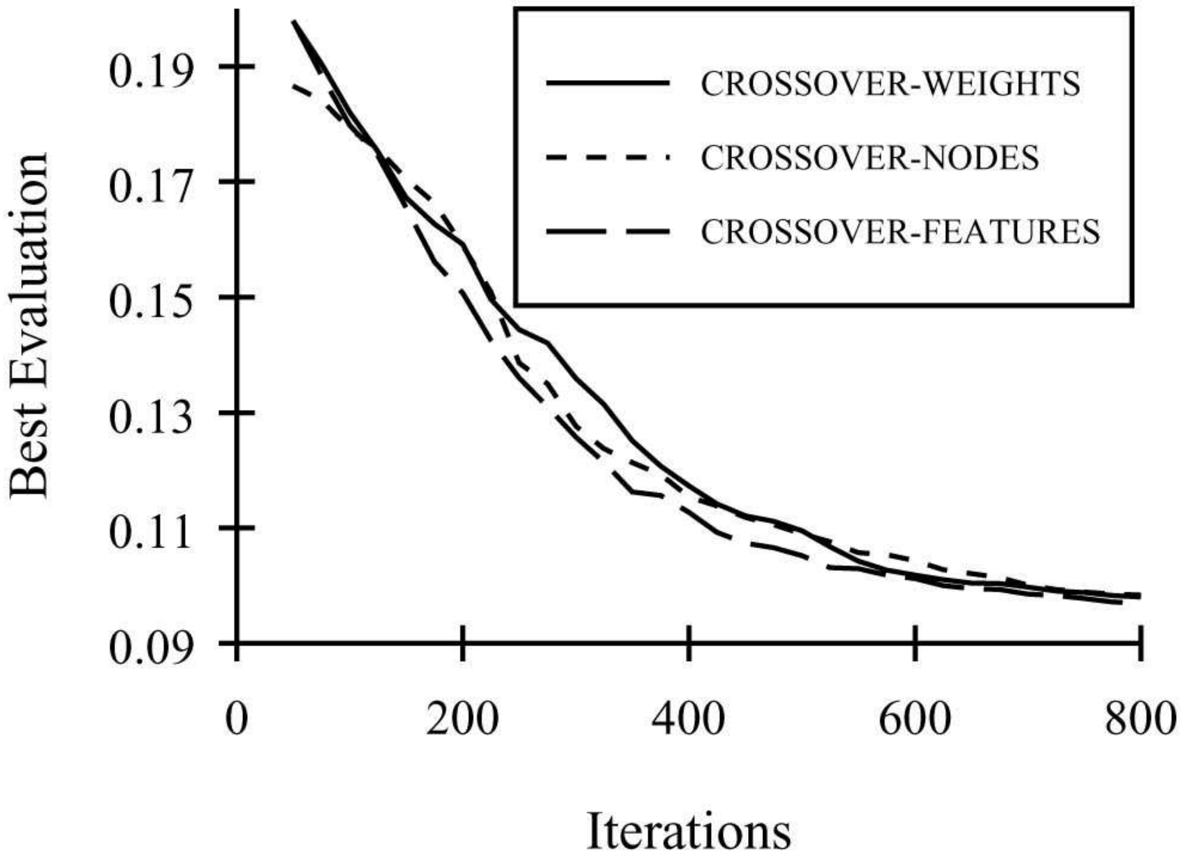
1. MUTATE-NODES
2. BIASED-MUTATE-WEIGHTS
3. UNBIASED-MUTATE-WEIGHTS

Experimental Results — Feedforward Sigmoidal



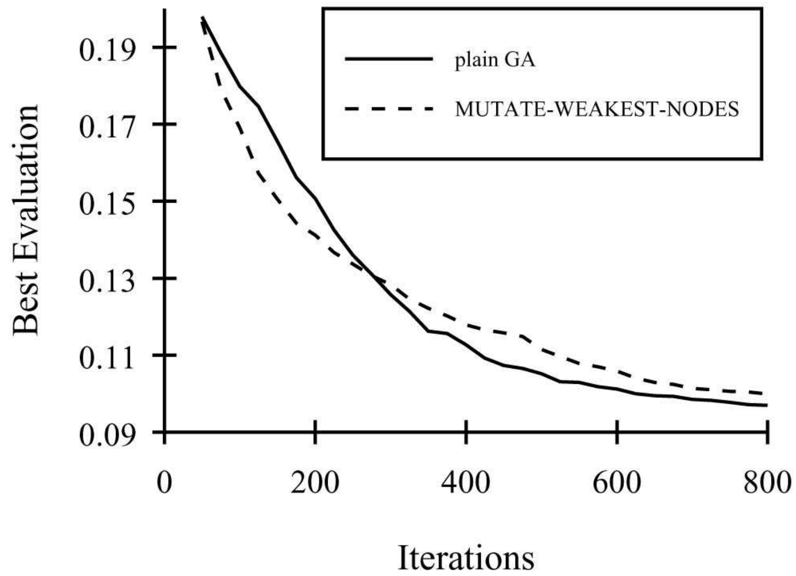
Experimental Results — Feedforward Sigmoidal (Cont.)

Second, a comparing of the three crossovers produced no clear winner.



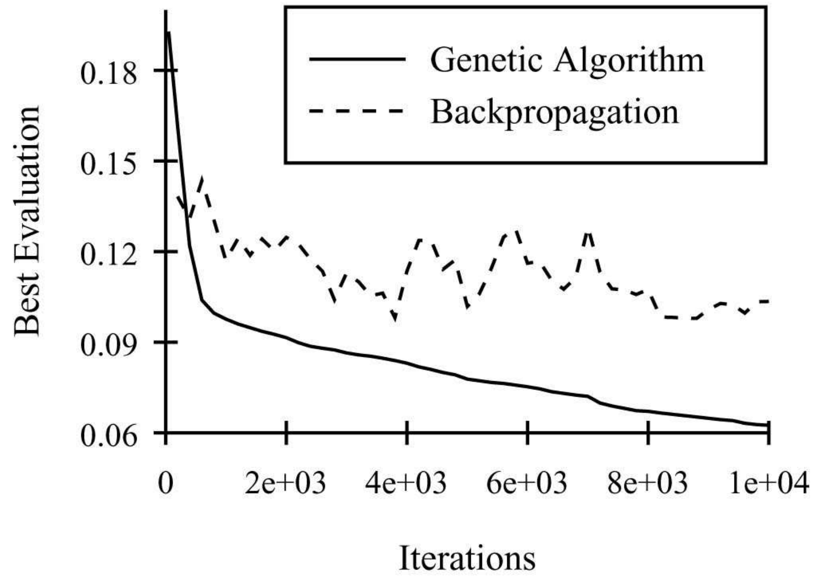
Experimental Results — Feedforward Sigmoidal (Cont.)

Third, when MUTATE-WEAKEST-NODE was added to a mutation and crossover operator, it improved performance only at the beginning. The performance decreased after a certain small amount of time.



Experimental Results — Feedforward Sigmoidal (Cont.)

Finally, a generic training algorithm with operators MUTATE-NODES and CROSSOVER-NODES outperformed backpropagation on our problem.



Experimental Results — Feedforward Sigmoidal (Cont.)

Backpropagation Algorithm is more computationally efficient than Genetic Algorithm.

Five Components of the genetic algorithm with WPNN:

Representation

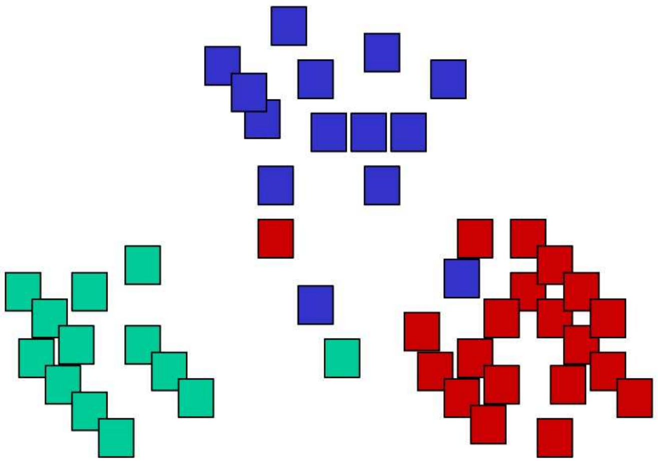
- a "logarithmic" representation for large dynamic range with proportional resolution;
- Map: $n \rightarrow B^{n-k_0}$

2. Evaluation Function

- "leaving-one-out" technique:
  a special form of cross-validation
- Performance function:

$$E = \sum_{i=1}^{M} \sum_{j=1}^{N_i} \left\{ \left[1 - \tilde{P}_i(\vec{x}_j^i) \right]^2 + \sum_{q \neq i} \left[\tilde{P}_q(\vec{x}_j^i) \right]^2 \right\}$$

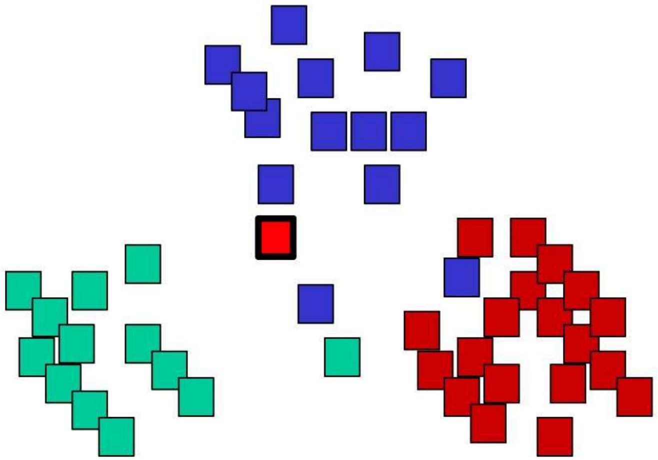
Leave-One-Out Method



For $k = 1, 2, \ldots, k$

$\mathrm{Err}(k) = 0$

Leave-One-Out Method

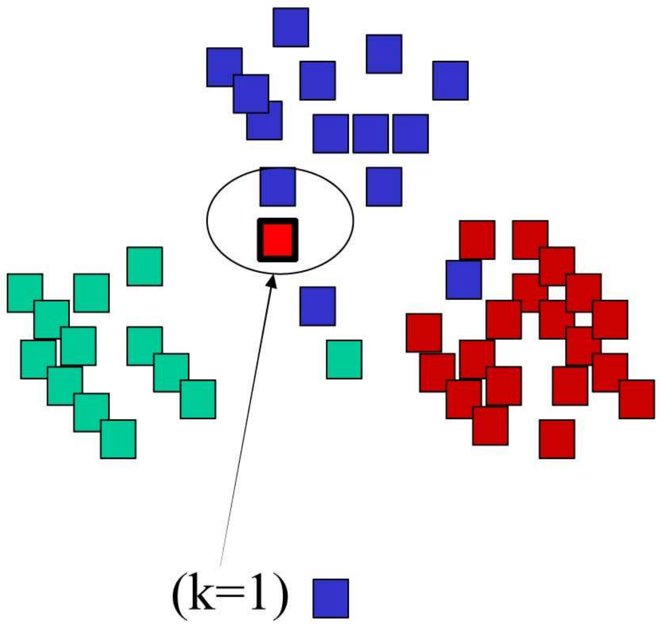


For $k = 1, 2, \ldots, k$

$\mathrm{Err}(k) = 0$

1. Randomly select a training data point and hide its class label

Leave-One-Out Method

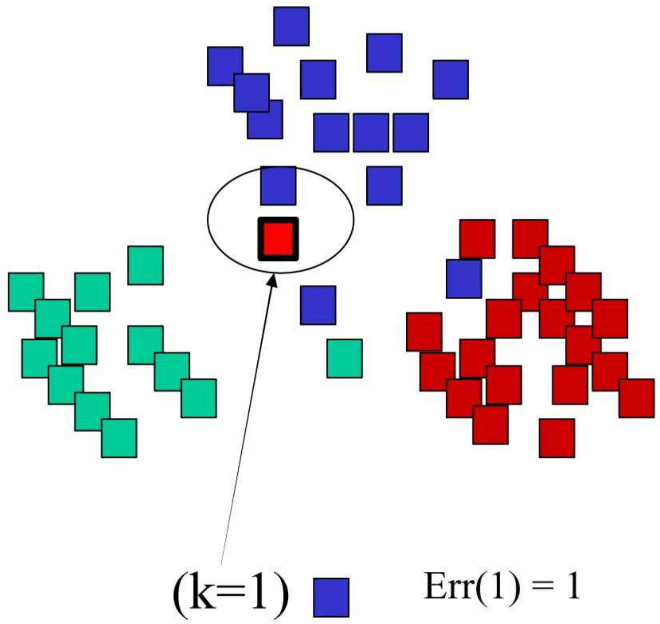


For $k = 1, 2, \ldots, k$

$\text{Err}(k) = 0$

1. Randomly select a training data point and hide its class label
2. Using the remaining data and given k to predict the class label for the left data point

Leave-One-Out Method



For $k = 1, 2, \ldots, k$

$\text{Err}(k) = 0$

1. Randomly select a training data point and hide its class label
2. Using the remaining data and given k to predict the class label for the left data point
3. Err(k) = Err(k) + 1 if the predicted label is different from the true label

Leave-One-Out Method

For $k = 1, 2, \ldots, k$

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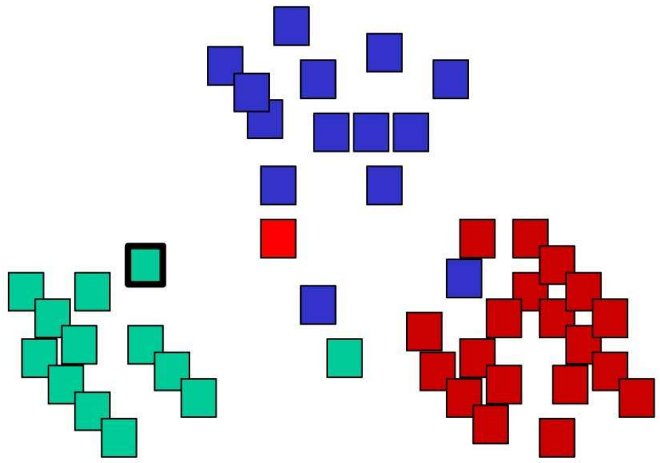
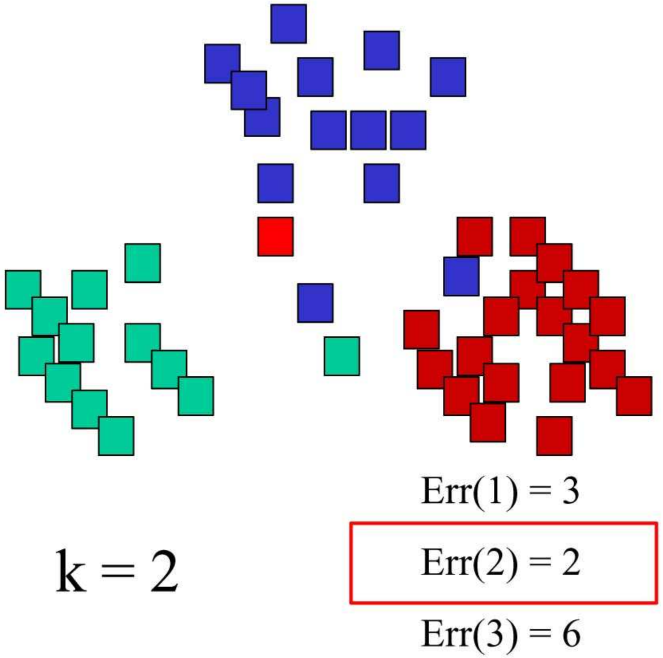


1. Randomly select a training data point and hide its class label
2. Using the remaining data and given k to predict the class label for the left data point
3. $\text{Err(k)} = \text{Err(k)} + 1$ if the predicted label is different from the true label

Repeat the procedure until all training examples are tested.

Leave-One-Out Method



For $k = 1, 2, \ldots, k$

$\mathrm{Err}(k) = 0$

1. Randomly select a training data point and hide its class label
2. Using the remaining data and given k to predict the class label for the left data point
3. Err(k) = Err(k) + 1 if the predicted label is different from the true label

Repeat the procedure until all training examples are tested.

Choose the k whose Err(k) is minimal

Empirical observations and theoretical arguments show WPNN works best when only a small fraction of the exemplars contribute significantly. So we reject a particular, for any exemplar x_j^i , if

$$\sum_{\vec{x}\neq\vec{x}_j} e^{-(\vec{x}-\vec{x}_j^i)^T \Sigma^{-1} (\vec{x}-\vec{x}_j^i)} > \Big(\sum_{i=1}^{M} N_i\Big)/P$$

Where $P = 4$

3. Initialization Procedure

WPNN: integers chosen randomly in $[1, K]$.

Where K depends on the desired range and resolution for weight

4. Genetic Operators

WPNN: standard GA mutation and *uniform crossover*

Uniform crossover example:

Individual	Genotype
`A`	`a b c d e f`
`B`	`q w e r t y`
`Offspring`	`q b e d t f`

The gene at locus j, where $0 \leq j <$ string length, from both parents have equal selected for the new offspring.

Why uniform?

- A real-valued representation is used and no particular
- ordering to the feature weights.

5. Parameter Values

- Population-Size: 1600
- Generation-Size:
  1. "steady-state" approach. With the large population a steady-state GA is used, which means Generation-Size is small relative to Population-Size.
  2. for using a single CPU, Generation-size = 1.
- Parent-Scalar: the smaller the Parent-Scalar, the faster the converge.
  Parent-Scalar = 0.9

Sample run of a steady-state GA

1. Initial population(randomly generated)
   (a) `(1001010) eval = 3`
   (b) `(0100110) eval = 3`
   (c) `(1101011) eval = 5`
   (d) `(0110101) eval = 4`
2. New children, from crossover or 3rd and 4th:
   (a) `(1101101) eval = 5`
   (b) `(0110011) eval = 4`

Experimental Results — WPNN

WPNN was a new and untested algorithm, the experiments with WPNN centered on the overall performance rather than on the training algorithm.

Experimental Results — WPNN (Cont.)

4 data set designed are used to illustrate both the advantages and shortcomings of WPNN.

Experimental Results — WPNN (Cont.)

First Data Set:

1. It is a training set that is generated during an effort to classify simulated sonar signals.
2. 10 features
3. 5 classes
4. 516 total exemplars

Experimental Results — WPNN (Cont.)

Second Data Set: Same as first data set except:

5 more features are added (which were random numbers uniformly distributed between 0 and 1 hence contained no information relevant to classification)

Experimental Results — WPNN (Cont.)

Third Data Set: Same as first data set except:

10 irrelevent features are added. (total of 20 features)

Experimental Results — WPNN (Cont.)

Fourth Data Set:

- Has 20 features just like the third data set.
- Pair each of the true feature with one of the irrelevant features.
- Mixing up the relevant features with the irrelevant features with via linear combinations.
- $0.5(f_i + g_i)$ and $0.5(f_i - g_i + 1)$

Experimental Results — WPNN (Cont.)

Dataset	1	2	3	4
Backpropagation	11	16	20	13
PNN	9	94	109	29
WPNN	10	11	11	25